Методы интегрирования.

1. Метод замены переменной (подстановка) – это путем введения новой переменной интегрирования, удается свести заданный интеграл к интегралу с новой переменной, который можно вычислить, используя непосредственное интегрирование.

<u>Вывод формул (наизусть!!!)</u>

$$1)\int (kx+b)^{\alpha}dx=\left|\begin{array}{l}kx+b=t\\(kx+b)'dx=t'dt\\kdx=dt\\dx=\dfrac{dt}{k}\end{array}\right|=\int t^{\alpha}\cdot\frac{dt}{k}=\frac{1}{k}\int t^{\alpha}dt=\frac{1}{k}\cdot\frac{t^{\alpha+1}}{\alpha+1}+C=\frac{(kx+b)^{\alpha+1}}{k(\alpha+1)}+C$$

$$2)\int \sin(kx+b)dx=\left|\begin{array}{l}kx+b=t\\(kx+b)'dx=t'dt\\kdx=dt\\dx=\dfrac{dt}{k}\end{array}\right|=\int \sin t\cdot\frac{dt}{k}=\frac{1}{k}\int \sin tdt=\frac{1}{k}\cdot\cos t+C=-\frac{\cos(kx+b)}{k}+C$$

$$3)\int \cos(kx+b)dx=\left|\begin{array}{l}kx+b=t\\(kx+b)'dx=t'dt\\kdx=dt\\dx=\dfrac{dt}{k}\end{array}\right|=\int \cos t\cdot\frac{dt}{k}=\frac{1}{k}\int \cos tdt=-\frac{1}{k}\cdot\sin t+C=\frac{\sin(kx+b)}{k}+C$$

$$4)\int \frac{dx}{\sin^2(kx+b)}=\left|\begin{array}{l}kx+b=t\\(kx+b)'dx=t'dt\\kdx=dt\\dx=\dfrac{dt}{k}\end{array}\right|=\int \frac{1}{\sin^2 t}\cdot\frac{dt}{k}=\frac{1}{k}\int \frac{1}{\sin^2 t}dt=-\frac{1}{k}\cdot ctgt+C=-\frac{ctg(kx+b)}{k}+C$$

$$5)\int \frac{dx}{\cos^2(kx+b)}=\left|\begin{array}{l}kx+b=t\\(kx+b)'dx=t'dt\\kdx=dt\\dx=\dfrac{dt}{k}\end{array}\right|=\int \frac{1}{\cos^2 t}\cdot\frac{dt}{k}=\frac{1}{k}\int \frac{1}{\cos^2 t}dt=-\frac{1}{k}\cdot tgt+C=\frac{tg(kx+b)}{k}+C$$

$$6)\int \frac{dx}{kx+b}=\left|\begin{array}{l}kx+b=t\\(kx+b)'dx=t'dt\\kdx=dt\\dx=\dfrac{dt}{k}\end{array}\right|=\int \frac{1}{t}\cdot\frac{dt}{k}=\frac{1}{k}\int \frac{1}{t}dt=\frac{1}{k}\ln|t|+C=\frac{\ln|t|}{k}+C$$

$$7)\int \frac{dx}{\sqrt{kx+b}}=\left|\begin{array}{l}kx+b=t\\(kx+b)'dx=t'dt\\kdx=dt\\dx=\dfrac{dt}{k}\end{array}\right|=\int \frac{1}{\sqrt{t}}\cdot\frac{dt}{k}=\frac{1}{k}\int \frac{1}{\sqrt{t}}dt=\frac{1}{k}\cdot 2\sqrt{t}+C=\frac{2\sqrt{kx+b}}{k}+C$$

$$8)\int a^{kx+b}dx=\left|\begin{array}{l}kx+b=t\\(kx+b)'dx=t'dt\\kdx=dt\\dx=\dfrac{dt}{k}\end{array}\right|=\int a^t\cdot\frac{dt}{k}=\frac{1}{k}\int a^t dt=\frac{1}{k}\cdot\frac{a^t}{\ln a}+C=\frac{a^{kx+b}}{k\ln a}+C$$

$$9)\int e^{kx+b}dx=\left|\begin{array}{l}kx+b=t\\(kx+b)'dx=t'dt\\kdx=dt\\dx=\dfrac{dt}{k}\end{array}\right|=\int e^t\cdot\frac{dt}{k}=\frac{1}{k}\int e^t dt=\frac{1}{k}\cdot e^t+C=\frac{e^{kx+b}}{k}+C$$

$$10)\int\frac{1}{\sqrt{a^2-x^2}}dx=\int\frac{1}{\sqrt{a^2\left(1-\frac{x^2}{a^2}\right)}}dx=\int\frac{1}{a\sqrt{\left(1-\left(\frac{x}{a}\right)^2\right)}}dx=\frac{1}{a}\int\frac{1}{\sqrt{\left(1-\left(\frac{x}{a}\right)^2\right)}}dx=\left|\begin{array}{l}\dfrac{x}{a}=t\\\left(\dfrac{x}{a}\right)'dx=t'dt\\\dfrac{1}{a}dx=dt\\dx=adt\end{array}\right|=$$

$$=\frac{1}{a}\int\frac{1}{\sqrt{\left(1-(t)^2\right)}}\cdot adt=\frac{a}{a}\int\frac{1}{\sqrt{1-t^2}}dt=\int\frac{1}{\sqrt{1-t^2}}dt=\arcsin t+C=\arcsin\frac{x}{a}+C$$

$$11)\int\frac{1}{a^2+x^2}dx=\int\frac{1}{a^2\left(1+\frac{x^2}{a^2}\right)}dx=\int\frac{1}{a^2\left(1+\left(\frac{x}{a}\right)^2\right)}dx=\frac{1}{a^2}\int\frac{1}{1+\left(\frac{x}{a}\right)^2}dx=\left|\begin{array}{l}\dfrac{x}{a}=t\\\left(\dfrac{x}{a}\right)'dx=t'dt\\\dfrac{1}{a}dx=dt\\dx=adt\end{array}\right|=\frac{1}{a^2}\int\frac{1}{1+t^2}\cdot adt=$$

$$=\frac{a}{a^2}\int\frac{1}{1+t^2}dt=\frac{1}{a}\int\frac{1}{1+t^2}dt=\frac{1}{a}arctgt+C=\frac{1}{a}arctg\frac{x}{a}+C$$

$$12)\int tgdx=\int\frac{\sin x}{\cos x}dx=\left|\begin{array}{l}\cos x=t\\(\cos x)'dx=t'dt\\-\sin xdx=dt\\\sin xdx=-dt\end{array}\right|=\int-\frac{dt}{t}=-\int\frac{dt}{t}=-\ln|t|+C=-\ln|\cos x|+C$$

$$13)\int ctgdx=\int\frac{\cos x}{\sin x}dx=\left|\begin{array}{l}\sin x=t\\(\sin x)'dx=t'dt\\\cos xdx=dt\end{array}\right|=\int\frac{dt}{t}=\ln|t|+C=\ln|\sin x|+C$$